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1977 J. Phys. A: Math. Gen. 10 1111

(http://iopscience.iop.org/0305-4470/10/7/009)

J. Phys. A: Math. Gen., Vol. 10, No. 7, 1977. Printed in Great Britain. © 1977

Massive spin-2 particles (*f*-gravity) in the Einstein space

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Received 7 January 1977, in final form 10 February 1977

Abstract. By using a variational principle, the equations of motion of massive spin-2 particles have been derived in the Einstein space.

1. Introduction

The experimental detection of spin-2 massive particles in nuclear interaction has led Isham *et al* (1971) to formulate a theory of strong or *f*-gravity for such a short-range interaction. This theory is along Einsteinian lines with a coupling constant which is assumed to be of the same order of magnitude as that for the strong interaction. Lord *et al* (1974) have proposed a model for strong gravity with a background de Sitter space instead of the usual flat space. They have taken the Einstein equations with the Λ term which, on linearization, yield Klein–Gordon type equations, and the Λ term serves as the mass term.

Recently, Mahanta (1976) has proposed a model for strong gravity wherein the equations are obtained from a variational principle using an approach due to Rosen (1966). In this paper we shall adhere to this approach and obtain the equations for spin-2 massive particles in the Einstein space.

Mahanta (1976) starts with

$$\delta \int I\sqrt{-g}\, \mathrm{d}^4x = 0 \tag{1}$$

where

$$I = P^{lijk}[R_{lijk} - \Lambda(g_{lj}g_{ik} - g_{lk}g_{ij})] + KL. \tag{2}$$

P^{lijk} is a fourth-rank tensor with all the symmetry properties of the curvature tensor R_{lijk}, L is the Lagrangian of matter and of all non-gravitational fields, while Λ and K are constants. Variation of P^{lijk} gives the space of constant curvature and variation of g_{ij} yields

$$P^{lijk}_{;ij} - \Lambda g_{ij}P^{lijk} = KT^{lk} \tag{3}$$

where K has been re-defined. Choosing

$$P^{lijk} = g^{lj}h^{ik} + g^{ik}h^{lj} - g^{lk}h^{ij} - g^{ij}h^{lk} \tag{4}$$

and suitable subsidiary conditions, he obtains the well known Dirac–Fierz–Pauli (Fierz and Pauli 1939) equations of spin-2 massive particles.

2. Massive spin-2 particles in the Einstein space

We shall, instead of (1), start with

$$\delta \int I\sqrt{-g}\, d^4x = 0 \tag{5}$$

where

$$I = h^{ij}(R_{ij} - \Lambda g_{ij}) + KL. \tag{6}$$

Here h^{ij} is the symmetric second-rank tensor (dynamical variables independent of g_{ij}) and $R_{ik} = g^{lj}R_{lijk}$. Variation of h^{ij} in (5) gives

$$R_{ij} = \Lambda g_{ij} \tag{7}$$

which is the Einstein space.

Variation of g_{ij} in (5) yields

$$\Box h^{ij} + h^{lk}_{;lk} g^{ij} - h^{lj}_{;kl} g^{ik} - h^{ik}_{;lk} g^{lj} - 2\Lambda h^{ij} = KT^{ij} \tag{8}$$

where a semicolon denotes covariant differentiation and $\Box$ is the curved-space box operator. It may be verified that divergence of the left-hand side of equation (8) vanishes identically as required by the Kraichnan (1955) generalization of the Eddington (1960) theorem.

If we now impose the four subsidiary conditions as

$$(h^{ij} + \tfrac{1}{2} h g^{ij})_{;j} = 0 \tag{9}$$

we can then write equation (8) in the form:

$$\Box h^{ij} - \tfrac{1}{2}\Box h g^{ij} + h_{;lk} g^{lj} g^{ik} + 2h^{lk} R^{i\ j}_{\ lk} = KT^{ij} \tag{10}$$

where equation (7) has also been used. On contraction the above equation gives

$$-2\Lambda h = KT. \tag{11}$$

h^{ij} has, in general, ten independent components of which only five now remain independent owing to the four subsidiary conditions (9) and the one condition implied by (11). Hence h^{ij} has the correct number of independent components for a spin-2 $(2s+1=5)$ particle field.

In empty space, $T^{ij} = 0$ which means $h = 0$, and so we have

$$\Box h^{ij} + 2h^{lk} R^{i\ j}_{\ lk} = 0 \tag{12}$$

together with

$$h^{ij}_{;j} = 0, \qquad h = 0. \tag{13}$$

This equation could be considered as the generalized Klein–Gordon equation for massive spin-2 particles in the Einstein space. A similar equation is obtained for g^{ij} by Sciama *et al* (1969) while considering the generally covariant integral formulation of Einstein's equations.

To write equation (12) in the Dirac–Fierz–Pauli form we shall further assume the space to have constant curvature, that is:

$$R_{lijk} = \tfrac{1}{3}\Lambda(g_{lj}g_{ik} - g_{lk}g_{ij}). \tag{14}$$

Now choosing the point of interest as the origin of the coordinate system, we can put equation (12) in the form (as in the case of Mahanta 1976):

$$\Box h^{ij} - \tfrac{2}{3}\Lambda h^{ij} = 0, \qquad h^{ij}_{;j} = 0, \qquad h = 0 \tag{15}$$

where $\Box$ is the ordinary box operator. Here Λ behaves as the mass term.

We may also note that by another suitable choice of subsidiary conditions we can obtain from equation (8) the equations for a spin-0 massive particle field.

3. Discussion

The novel features of this approach are: (i) both the geometry and the field equations follow from the same variational principle; (ii) the variational function does not contain first derivatives of the dynamical variables (h^{ij}); (iii) the inertia of particles is related to the universe at large through Λ which serves as the mass term in equation (15); and (v) the (non-linearized) equations are exact.

Our variational function (6) is simpler than that of Mahanta (1976) since it involves only the second-rank tensors and hence we do not have to assume a relation like (4). The geometry of space (Einstein space) is more general though we have to assume the constant curvature of space so as to put the equations in the Dirac–Fierz–Pauli form.

Besides the equations (12) and (13) perhaps serving as a model for strong gravity with the Einstein space as the background, they are interesting of their own accord and as such call for attention.

It has also been shown elsewhere (Mahanta and Dadhich 1976) that one can always construct the energy–momentum complex in such theories involving dynamical variables on similar lines as in the Einstein theory.

Acknowledgments

I wish to thank Dr M N Mahanta, Indian Institute of Technology, Delhi, who introduced me to this problem, for persistent helpful discussions. I should like to thank Professor N Mukunda, Chairman, Centre for Theoretical Studies, Indian Institute of Science, Bangalore, for general discussions and hospitality at the Centre where the work was initiated. Thanks are also due to Professor J V Narlikar, Tata Institute for Fundamental Research, Bombay for reading the first write up of the paper.

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